Series 7 (3.11.2011)

Submission: **15:00, November, 10, 2011**, into the boxes next to the room I-21. Write every solution on the separate paper (format A4)! Don't forget to sign your solutions.

It is not sufficient to answer a single number or yes/not. Answers always have to be justified.

Exercise 1. Let $L_1, L_2 \subseteq \{a,b\}^*$ are languages and $|L_1| = 4$, $|L_2| = 7$. What is maximal/minimal value of:
a) $|L_1 \cup L_2|$
b) $|L_1 \cap L_2|$
c) $|L_2 \setminus L_1|$
Prove your claims.

Exercise 2.
a) Design an algorithm which decides for given regular expression E over the alphabet Σ and a word $w \in \Sigma^*$ if $w \in L(E)$. Our algorithm should not execute more operations than $O(|w||E|^2)$, where $|E|$ is the length of the expression E.
b) Regular expressions are closed not only under the operations of union, concatenation and iteration but they are closed under many other operations too. For example under intersection, difference, complement and other. Generalized regular expressions can contain intersection and complement plus union, concatenation and iteration. Consider alphabet $\Sigma = \{a,b,c\}$. Expres the generalized regular expressions $\{abc\}^C$ and $(\Sigma^* abc \Sigma^*)^C \cap (\Sigma^* cba \Sigma^*)^C$ as ordinary regular expressions (e.g. without use of complement and intersection).

Exercise 3. Modular construction produces from two DFA A_1 and A_2 such DFA A that $L(A) = L(A_1) \cap L(A_2)$ or $L(A) = L(A_1) \cup L(A_2)$.

In case we have two NFA B_1, B_2 recognizing $L(B_1)$ and $L(B_1)$ respectively and we want to construct an automaton B which recognizes $L(B_1) \cap L(B_2)$ or $L(B_1) \cup L(B_2)$, we can transform both B_1 and B_2 into deterministic automata and then use modular construction. The automaton produced in this way can have exponential number of states in respect to n_1, and n_2, where automata B_1, B_2 had $n_1 = |Q_{B_1}|$ and. $n_2 = |Q_{B_2}|$ states.

Design general method for producing one NFA B from B_1 and B_2, which will accept union/itersection of the languages $L(B_1)$ and $L(B_2)$ in such a way that the incerase of the number of states will be at most polynomial in n_1 a n_2.