Series 1 (23.9.2011)

Submission: **15:00, September, 29, 2011**, in to the boxes next to the room I-21. Write every solution on the separate paper (format A4)! Don't forget to sign your solutions.

It is not sufficient to answer a single number or yes/not. Answers always have to be justified.

Exercise 1.

a) Find a language $L \neq \{\lambda\}$ such that for all $i \geq 1$ it holds $L^i = L$. Argue why your claim should hold. Is there a finite language L which satisfies this condition?

b) Describe the language $L = \{0^{m+2}1^m2^{2m} | m \geq 1\}$ as intersection of two languages L_1 and L_2, which satisfies following conditions:
   (i) $L_1 \not\subseteq L_2$ a $L_2 \not\subseteq L_1$,
   (ii) $L_1, L_2 \subseteq \{0\}^+\{1\}^+\{2\}^+$,
   (iii) $L_1 - L$ a $L_2 - L$ are infinite languages.

Exercise 2.

Prove or disprove the following claims:

a) $(\{a\}^*\{b\}^*)^* = (\{a,b\}^*)^2$,

b) $(\{a\}^*\{b\}^*)^* = (\{a,b\}^2)^*$.

Exercise 3.

a) Given word w of length k over an arbitrary alphabet Σ. How many different prefixes of w can there exists which are suffixes of w at the same time? Give the maximal and minimal number for any such word w, depending on k. Hint: Consider in each case an appropriate aplhabet.

b) For the case $k = 5$ and the alphabet $\Sigma = \{a, b\}$ give two words over Σ, in both of which both letters of Σ should appear at least once. In one word, the number of prefixes that are suffixes, too, shall be as large as possible, in the other one it shall be as small as possible.